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STABILITY OF AXIALLY SYMMETRIC DEFORMATIONS OF SPHERICAL SHELLS UNDER AXIALLY SYMMETRIC LOAD

by

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DRAFT TRANSLATION

STABILITY OF AXIALLY SYMMETRIC DEFORMATIONS OF SPHERICAL SHELLS UNDER AXIALLY SYMMETRIC LOAD* [Russian title]*

Transl. ... from

Doklady A. N. SSSR,
Teoriya Uprugosti (Moscow) 11 Aug. 1963
(Theory of Elasticity)
T. 151, No. 5, p. 1053-5
Moscow, August 11, 1963

by A. V. Pogorelov Sept. 1963
6 p refs

When investigating supercritical deformations of spherical shells under load and having an axial symmetry, the deformations are also assumed axially symmetric. However, experiments show that this may eventually not take place. Thus, for instance, if a freely-resting at its edge spherical segment is loaded by a concentrated force, applied at its center, the bulging region has a shape of a round meniscus at comparatively low value of this force. When the force attains a certain critical value, the meniscus' boundary begins to take shape of a triangle with rounded apexices (Fig. 1).

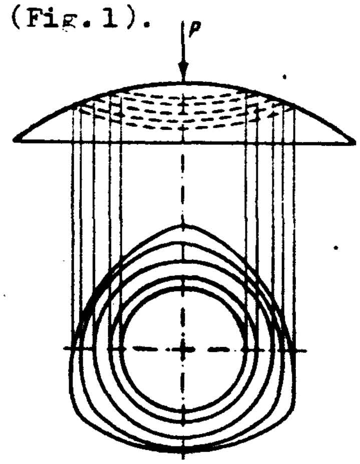

Fig. 1

Another example: When testing spherical shells for steadiness under external pressure, the hollow after " floc " at the surface of the sample has the shape of a polygon and not a circle, although both, the shell and the loading were characterized by perfect symmetry.

We shall present in the current note some results concerning the stabity of axially symmetric deformations of axially symmetric shells for two loading systems: concentrated force and uniform outer pressure.

* [OB USTOYCHIVOSTI OSESIMMETRICHESKIKH DEFORMATSIY SFERICHESKIKH OBOLOCHEK PRI OSESIMMETRICHESKIKH NAGRUZHENIYAKH.]

[redacted]

The general considerations relative to the investigation method have been expounded by the author in preceding publications and namely in the work under reference [1].

1.- First of all we wish to point out the very possibility of disruption of axial symmetry of deformations. The fact is that the supercritical deformation of a strictly convex shell, fastened at its edge is obtained in a known approximation by specular reflection of a certain of its segments, as was shown in [1]. The demonstration of this assertion is based upon two positions :

1) elastic deformations of a shell represent basically a geometrical bending;

2) a strictly convex surface fastened at the edge does not admit bendings without regularity disruption. The application of this result to real shells is limited by two conditions, corollary of premises 1) and 2). Namely, the bulging region must occupy a substantial part of its surface, so that the geometrical condition of its edge fastening be not weakened by the deformation of the median surface of the shell. The stresses in shell's material must not exceed the limit of elasticity, in order that the relative deformations of the median surface could be estimated small. In both examples presented of the disruption of spherical shell's axially symmetric deformations, these conditions are not satisfied. That is why the very deflection of the deformation from axial symmetry are not in contradiction with the results obtained in the work [1].

2.- The state of elastic equilibrium of the shell is defined by minimizing the functional

$$W(F) = U(F) - A(F),$$

which is considered in isometrical transformations of the initial surface of the shell. $U(F)$ is the energy of shell's deformation into shape F; $A(F)$ is the work accomplished at that external load.

The energy U(F) of the shell along the main surface of the shell (excluding the rib) is determined by the standard method, and along ribs — by the formula

$$U = \int_{\gamma} c\, E \delta^{5/2} \alpha^{5/2} k^{1/2}\, ds$$

(see [1]). Here 2α is the angle between the tangents to surface F planes along the rib γ; k — is the rib's curvature; δ - is the thickness of the shell; the integrating is effected along rib's γ arc. The constant $c \approx 0.18$. The main difficulty of solving this problem of elastic equilibrium of the shell when using this approach is reduced to the determination of all isometric transformations of shell's median surface.

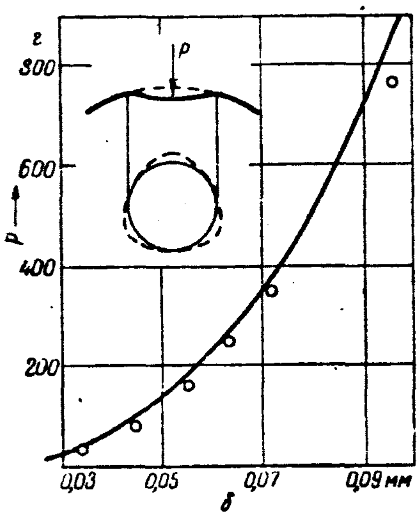

Fig. 2

3. — We limit ourselves to considering those isometric transformations of the spherical surface, at which a rib γ is formed along the curve given by the equation

$$r = R\rho\,(1 + \lambda \cos k\vartheta)$$

where r, ϑ are the polar geodesic coordinates. The surface with such a rib, isometric to the sphere, will depend on two parameters ρ and λ. We find the shape of such surface at sufficiently small values of ρ and λ. Further, we find in the assumption of such smallness of parameters ρ and λ, U(F) and A(F). For deformation's energy we obtain the expression

$$U(F) = 2\pi c\, E \delta^{5/2} R^{1/2} \rho^3 \left(1 + \lambda^2 \left(k^2 + \frac{(1+k^2)^2}{16}\right)\right) + \frac{\pi \rho^2 E \delta^3}{3} \lambda^2 (k^2 - 1)\; k \,.$$

The work A, effected by external load under the action of the concentrated force f is

../..

$$A(F) = f\rho^2\left(1 + \frac{\lambda^2 k^2}{2}\right)R.$$

Only the principal terms by order of magnitude are written in the expressions for U (F) and A (F).

For a shell in equilibrium condition, parameters ρ and λ, characterizing the deformation, are determined from the system of equations

$$\frac{\partial}{\partial \rho}(U - A) = 0, \quad \frac{\partial}{\partial \lambda}(U - A) = 0.$$

For a fixed k this system relative to ρ and λ always has the solution $\lambda = 0$ (axial-symmetric deformation). If ρ is sufficiently small, this solution will be unique. This means that at small deformation the bulging region has the shape of a circle. For great deformations (i. e. at great ρ), the system admits to the contrary the solution $\lambda \neq 0$. The value ρ limiting both these cases, satisfies the system of equations

$$\frac{\partial}{\partial \rho}(U - A)\big|_{\lambda=0} = 0, \quad \frac{1}{\lambda}\frac{\partial}{\partial \lambda}(U - A)\big|_{\lambda=0} = 0.$$

At $k = 3$, we hence obtain

$$\rho = \frac{1}{c}\sqrt{\frac{\delta}{R}},$$

or, introducing the radius $r = R\rho$ of the bulging circle

$$r = \frac{1}{c}\sqrt{\delta R}.$$

Thus, THE BULGING IN THE SHAPE OF A CIRCLE UNDER THE ACTION OF A CONCENTRATED FORCE IS STEADY SO LONG AS THE RADIUS OF THIS CIRCLE IS $r \leqslant \sqrt{\delta R}/c$. The critical force f is determined by the formula

$$f = \frac{3\pi E\delta^3}{R}.$$

4. We presented in Fig. 2 a graph of critical force f dependence on the thickness δ for copper shells of radius R = 80 mm. Dots indicate the values of the critical force obtained in the experiment.

5. — The investigation of supercritical deformations of a spherical shell at uniform external pressure leads to a similar conclusion as in 3. - for a concentrated load. Namely, the supercritical deformation has an axial symmetry, i.e. the bulging region has the shape of a circle so long as the radius of the circle $r \leqslant \sqrt{\delta R}/c$. After that the bulging region begins to take shape of a triangle with rounded up apexices.

***** THE END *****

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